NOVA SCHOOL OF BUSINESS & ECONOMICS

Mathematics Written Exam

Date	Duration + Tolerance	Elements of consultation allowed
26 May 2022	2 hours + 15 minutes	Calculator

Instructions

The exam is composed by two compulsory groups.

Group I is composed by multiple choice questions and Group II is composed by open answer questions.

For each multiple choice question, four answer options are presented, of which exactly one is correct. Indicate your answer by selecting the letter corresponding to the option that you consider to be correct. Each correct, incorrect or null answer in Group I has the classification of, respectively, 1/20, −0.33/20 and 0/20 points. In case more than one option is selected for the same question, the answer will be considered null. A negative total for this set of questions adds zero to the final grade.

All open answer questions should be carefully justified, and all computations should be presented. The classification for each question in this group is indicated next to it.

Do not unstaple this book.

Do not use any type of corrector. If necessary, cross out.

Name of the Candidate

Group I

1 (1/20) Consider, in an orthonormal referential $Oxyz$, the plane α and the line r defined by

$$\alpha: x - y + z = 0, \qquad r: \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z}{3}.$$

Which of the following propositions is true?

a) The line r is contained in the plane α.

b) The point of intersection of α and r is $\left(0, \frac{3}{2}, \frac{3}{2}\right)$.

c) The plane α and the line r do not intersect.

d) None of the above.

2 (1/20) Which of the following conditions defines, on the complex plane, a perpendicular line to the imaginary axis?

a) $|3z - 2i| = 5$

b) $z + \bar{z} = 0$

c) $\left|z + i\sqrt{\pi}\right| = |z - ie^2|$

d) None of the above.

3 (1/20) What is the maximal domain of the function f, of real variable, given by

$$f(x) = \frac{-\pi\sqrt{e-x}}{\ln(1-x^2)\sqrt[3]{2-e^{-x}}}?$$

a) $]-1, e[\setminus\{-\ln(2), 0, 1\}$

b) $]\ln(2), e[\setminus\{1\}$

c) $]-1, 1[\setminus\{-\ln(2), 0\}$

d) None of the above.

4 (1/20) Suppose that (u_n) is a geometric progression with $u_1 = 2$ and $u_4 = -16$. Which is the sixth term of (u_n)?

a) -64

b) -32

c) 12

d) None of the above.

5 (1/20) Consider the sequence (u_n) with general term

$$u_n = \begin{cases} \sqrt{2^{-n}} + e^2 \left(\frac{n^2+1}{n^2+3}\right)^{n^2}, & \text{if } n \text{ even} \\ \frac{n^3+5n+1}{n^3+3} - \frac{6}{\ln\left(\frac{1}{n^2+5}\right)}, & \text{if } n \text{ odd} \end{cases}.$$

Which of the following propositions is true?

a) $\lim u_n = 0$

b) $\lim u_n = 1$

c) The sequence (u_n) does not admit limit.

d) None of the above.

6 (1/20) A bag contains five cards, numbered from 1 to 5. Rita will successively, and randomly, extract the five cards from the bag, and will align them, from left to right, according to the order of extraction, in order to produce a five digit number. What is the probability that the number is even and has an odd tens digit?

a) $\frac{3}{10}$

b) $\frac{5}{36}$

c) $\frac{5}{108}$

d) None of the above.

7 (1/20) A given electronic system is composed by two subsystems, A and B. From previous tests, it is known that the probability that A fails is 0.2, the probability that B fails and A does not fail is 0.15, and the probability that A and B fail simultaneously is 0.15. What is the probability that neither A nor B fail?

a) 0.35

b) 0.65

c) 0.95

d) None of the above.

Group II

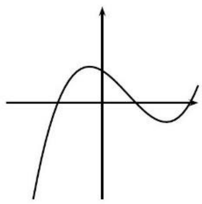
1 (1/20) Let f be a function with domain $\mathbb{R}$ and image $[-4,1]$. What is the image of the function $-|f|+2$? Justify your answer.

2 (1/20) Let $f\colon \mathbb{R}\setminus\{-1,1\} \longrightarrow \mathbb{R}$ be the function defined by

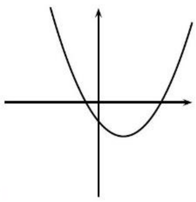
$$f(x) = \frac{x^2-1}{|x|-1}.$$

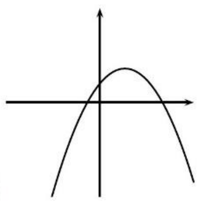
Using exclusively analytical methods, find $\lim_{x\to 1} f(x)$ e $\lim_{x\to -1} f(x)$, in case they exist. Present all your computations.

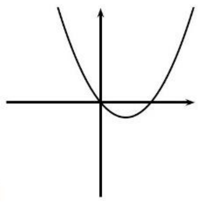
3 (1/20) The figure below represents the graph of a differentiable function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$:



In which of the following figures can a graphic representation of the derivative of f be found? Justify your answer.


(A)


(B)


(C)

4 (2/20) Suppose that the line defined by $2x+y=1$ is tangent to the graph of $f\colon \mathbb{R} \longrightarrow \mathbb{R}$, at the point $(0,1)$, and that $g\colon \mathbb{R} \longrightarrow \mathbb{R}$ is such that

$$g(x) = \sqrt[3]{f(x)} - \frac{f(x)}{2+e^x},$$

for all $x \in \mathbb{R}$. Find the value of $g'(0)$, presenting all your computations.

5 (1/20) Using exclusively analytical methods, show that the equation

$$2\cos(x) = x^2$$

has solution in the interval $\left[0, \frac{\pi}{2}\right]$.

6 Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by

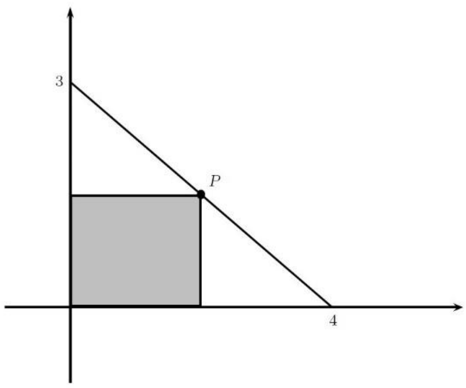
$$f(x) = \begin{cases} e^{x^2} \ln(-x + e + 1), & \text{if } x \in]0,1] \\ (2e - 1)(x - 1) + e, & \text{otherwise} \end{cases}.$$

Using exclusively analytical methods:

a) (2/20) Find $f'(1)$. Present all your computations.

b) (1/20) Find the equation of the tangent line to the graph of f at the point $x = 1$.

7 (2/20) Find the coordinates of the point P that maximize the area of the rectangle in the figure below:



Present all your computations.

8 (2/20) Using exclusively analytical methods, find the set of all $x \in \mathbb{R}$ that satisfy

$$-e^{2-x} \frac{(x-2)^3}{x^2+x-6} \ln(x + 5) < 0,$$

carefully justifying your answer and presenting all your computations. Present the solution set as a union of intervals.

THE END

Formulae

Special Limits

$$
\begin{aligned}
\lim\left(1+\frac{1}{n}\right)^n &= \mathrm{e} \qquad (n \in \mathbb{N})\\
\lim_{x\to 0}\frac{\operatorname{sen}(x)}{x} &= 1\\
\lim_{x\to 0}\frac{e^x-1}{x} &= 1\\
\lim_{x\to 0}\frac{\ln(x+1)}{x} &= 1\\
\lim_{x\to +\infty}\frac{\ln(x)}{x} &= 0\\
\lim_{x\to +\infty}\frac{e^x}{x^p} &= +\infty \qquad (p \in \mathbb{R})
\end{aligned}
$$

Derivation Rules

$$
\begin{aligned}
(u+v)' &= u'+v'\\
(uv)' &= u'v+uv'\\
\left(\frac{u}{v}\right)' &= \frac{u'v-uv'}{v^2}\\
(u^n)' &= nu^{n-1}u' \qquad (n \in \mathbb{R})\\
(a^u)' &= \ln(a)\, a^u u' \qquad (a \in \mathbb{R}^+ \setminus \{1\})\\
(\log_a(u))' &= \frac{u'}{\ln(a)\, u} \qquad (a \in \mathbb{R}^+ \setminus \{1\})\\
(\operatorname{sen}(a))' &= a' \cos(a)\\
(\cos(a))' &= -a' \operatorname{sen}(a)
\end{aligned}
$$