NOVA SCHOOL OF BUSINESS & ECONOMICS

Mathematics Written Exam

Date	Duration + Tolerance	Elements of consultation allowed
17 June 2025	2 hours + 15 minutes	Calculator

Instructions
This exam is composed of two compulsory groups. Group I is composed of multiple choice questions and Group II is composed of open-ended questions. For each multiple choice question, four answer options are presented, of which exactly one is correct. Indicate your answer by selecting the letter corresponding to the option that you consider to be correct. Each correct, incorrect or blank/null answer in Group I has the classification of, respectively, 1/20, −0.3/20 and 0/20 points. In case more than one option is selected for the same question, the answer will be considered null. The minimum grade in Group I is 0. All open-ended questions should be carefully justified, and all computations should be presented. The classification for each question in this group is indicated next to it. Do not unstaple this booklet. Do not use any type of corrector. If necessary, cross out.

Name of the Candidate

Group I

1 (1/20) Four girls and four boys enter a bus, in which there are six unoccupied seats. In how many different ways can these six seats be occupied by the group of girls and boys, assuming that two boys are left standing?

a) 3560

b) 3840

c) 4180

d) 4320

2 (1/20) Let Ω be the sample space of some random experiment and let A and B be two events in Ω. Suppose that

$$P(A) = 0.3 \quad \text{and} \quad P(B) = 0.5.$$

Which of the following can be the value of $P(A \cup B)$?

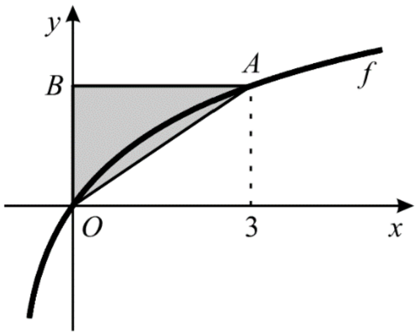
a) 0.4

b) 0.6

c) 0.9

d) None of the previous options.

3 (1/20) The figure below shows, on an orthonormal coordinate system xOy, part of the graph of a function f, defined by $f(x) = \log_2(x+1)$.



In the same figure, a right-angled triangle $[ABO]$ is represented. The point A has abscissa 3 and belongs to the graph of f. The point B belongs to the axis Oy.

What is the area of the triangle $[ABO]$?

a) 2

b) 3

c) 4

d) None of the previous options.

4 (1/20) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be the function defined by $f(x) = 3e^{1-2x^2} + 7$. What is the value of

$$\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}?$$

a) $4e^3$

b) $3e^{-1}$

c) $\frac{12}{e}$

d) None of the previous options.

5 (1/20) What is the general term of the sequence

$$1, 1+2, 1+2+3, 1+2+3+4, \ldots?$$

a) $u_n = 2n - 1$

b) $u_n = 3n - 2$

c) $u_n = \frac{n(n+1)}{2}$

d) None of the previous options.

6 (1/20) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$

Which of the following statements is true?

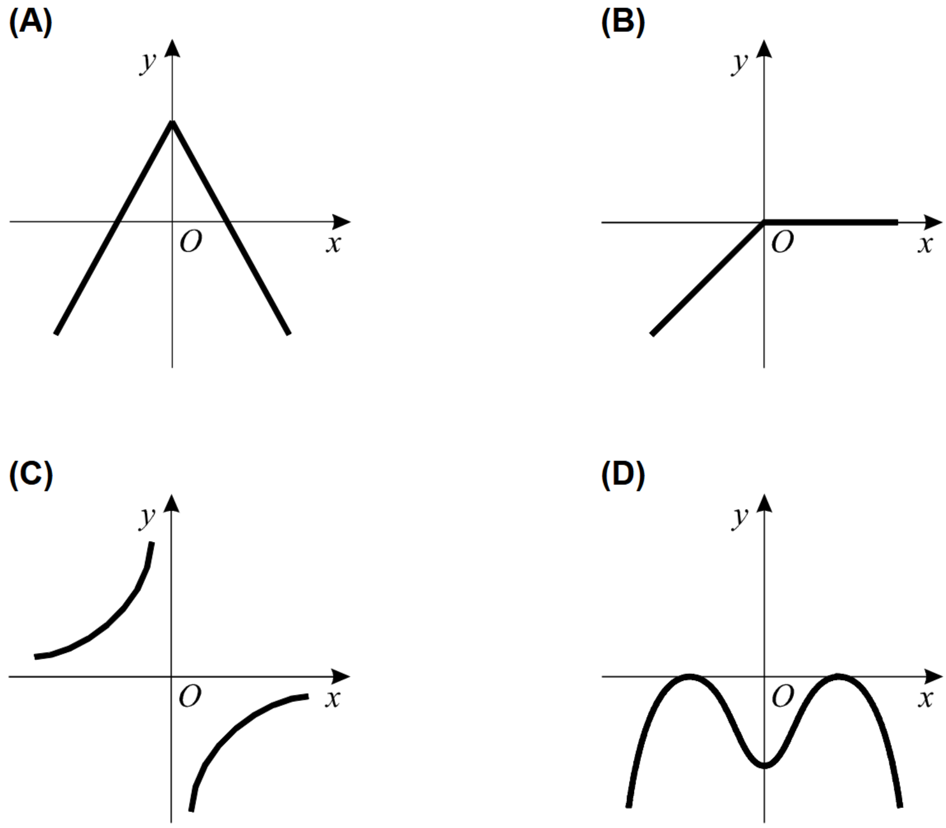
a) f is not continuous at 0.

b) $\lim_{x \to 0} f(x) = 0$.

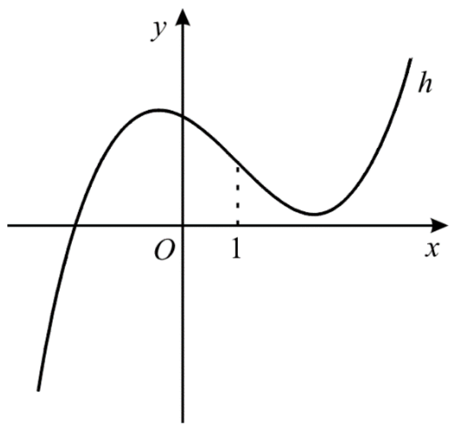
c) $f'(0) = 0$.

d) None of the previous options.

7 (1/20) In which of the following figures can we find part of the graph of an even function, with domain $\mathbb{R}$ and range $]-\infty, 0]$?



8 (1/20) The figure below shows, on an orthonormal coordinate system xOy, part of the graph of a polynomial function f. The point with abscissa 1 is the only inflection point of f.



Which of the following expressions can define f'', the second order derivative of f?

a) $f''(x) = (1-x)^2$

b) $f''(x) = x^2 - 1$

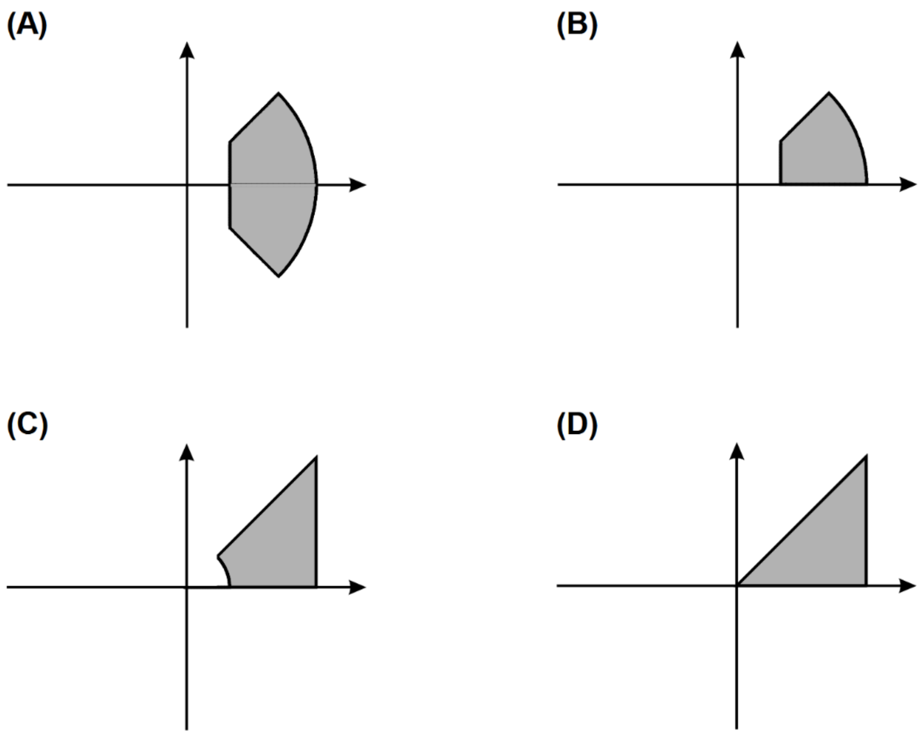
c) $f''(x) = 1 - x$

d) None of the previous options.

9 (1/20) Consider, in $\mathbb{C}$, the condition

$$|z| \leq 3 \quad \wedge \quad 0 \leq \arg(z) \leq \frac{\pi}{4} \quad \wedge \quad Re(z) \geq 1.$$

In which of the following figures can be represented, on the complex plane, the set of points defined by this condition?



10 (1/20) Which of the following propositions is true?

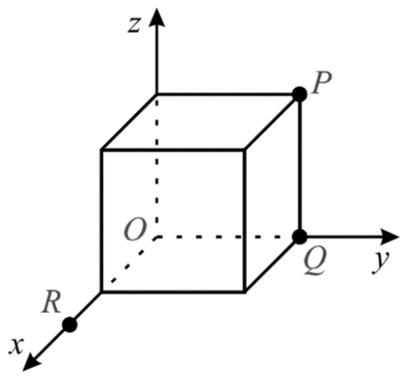
a) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ :\ \frac{x}{y} \neq 0$

b) $\exists x \in \mathbb{Z} : \forall y \in \mathbb{Z},\ x \leq y$

c) $\exists x, y \in \mathbb{R} \setminus \{0\} : x^2 + y^2 = 0$

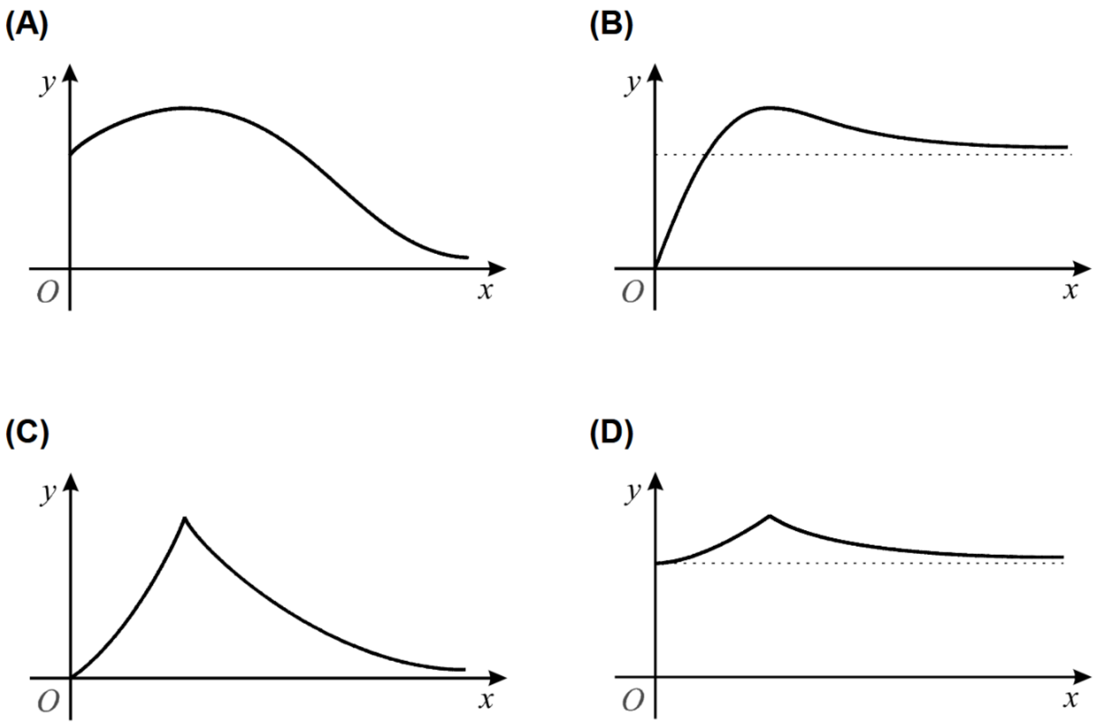
d) $\forall x \in \mathbb{R}^+\ \exists y \in \mathbb{R}^-\ :\ |x| - |y| = 0 \wedge xy < 0$

11 (1/20) In the figure below, a cube is represented, on an orthonormal coordinate system $Oxyz$.



Three of the cube's edges are contained in the coordinate system axes. The points P and Q are two of the cube's vertices, belonging to the plane yOz. Suppose that, starting from the origin of the coordinate system, a point R moves along the positive semiaxis Ox. Let f be the function that maps the abscissa of the point R to the area of the cross-section produced in the cube by the cut by the plane PQR.

Which of the following can be the graph of f?



12 (1/20) Of a function f, continuous in $\mathbb{R}$, it is known that $f(2) = 6$ and $f(8) = 1$. Which of the following propositions is necessarily true?

a) $1 \leq f(4) \leq 6$

b) $f(4) > f(5)$

c) The function f has no zeros in $[2,8]$.

d) 4 belongs to the range of f.

Group II

1 (2.5/20) Find, in case it exists, the limit of the sequence (u_n) with general term

$$u_n = \begin{cases} \dfrac{\pi n + n^2}{-n^2 - e^{-n}} . (-1)^{n+2} + n\, sin\left(\dfrac{1}{n}\right), & \text{if } n \text{ odd} \\ -2\ln\left(\left(\dfrac{4n-3}{4n+1}\right)^n\right) + \cos\left(\dfrac{\pi}{2} + 2\pi n\right), & \text{if } n \text{ even} \end{cases}$$

justifying your reasoning.

2 (2.5/20) Find the domain of the function f, of a real variable, defined by

$$f(x) = \frac{\sqrt[4]{-\ln\left(\left(\frac{1}{4}\right)^x\right)} . \sqrt{x^2 - 2x}}{\sqrt[3]{x-3} \ . \ \sqrt{e^{-x}} \ . \ |5 - x|},$$

justifying your reasoning and presenting all your computations.

3 (3/20) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be a function with real derivative in every point of its domain. Suppose that the straight line with equation $x + y = 0$ is tangent to the graph of f at the point with abscissa 0. Let $g: \mathbb{R} \longrightarrow \mathbb{R}$ be the function defined by

$$g(x) = xe^{x^3} + \text{sen}^3\big(f(x)\big) + \frac{2f(x)}{x^2+1} + \sqrt{x^4+1} + \ln\left(\big(f(x)\big)^2 + 1\right) + 3,$$

for all $x \in \mathbb{R}$. Find the value of $g'(0)$, presenting all your computations.

THE END

Formulae

Progressions

Sum of the first n terms of a progression (u_n):

Arithmetic progression: $\frac{u_1+u_n}{2} \times n$
Geometric progression: $u_1 \times \frac{1-r^n}{1-r}$

Geometry

Area of the disk: πr^2

Area of the triangle: base × height × $\frac{1}{2}$

Derivative Rules

$$(u+v)' = u' + v'$$
$$(u\,v)' = u'\,v + u\,v'$$
$$\left(\frac{u}{v}\right)' = \frac{u'\,v - u\,v'}{v^2}$$
$$(u^n)' = n\,u^{n-1}\,u' \quad (n \in \mathbb{R})$$
$$(\operatorname{sen} u)' = u' \cos u$$
$$(\cos u)' = -u' \operatorname{sen} u$$
$$(\operatorname{tg} u)' = \frac{u'}{\cos^2 u}$$
$$(e^u)' = u'\,e^u$$
$$(a^u)' = u'\,a^u \ln a \quad (a \in \mathbb{R}^+ \setminus \{1\})$$
$$(\ln u)' = \frac{u'}{u}$$
$$(\log_a u)' = \frac{u'}{u \ln a} \quad (a \in \mathbb{R}^+ \setminus \{1\})$$

Special Limits

$$\lim \left(1 + \frac{1}{n}\right)^n = e \quad (n \in \mathbb{N})$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Solutions

Group I

1. d)
2. b)
3. b)
4. c)
5. c)
6. a)
7. d)
8. d)
9. b)
10. d)
11. d)
12. d)

Group II

1. 2
2. $(\{0\} \cup [2, +\infty[) \setminus \{3,5\}$
3. -1